The mixing of ground water and sea water in permeable subsoils

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SUMMARY

The subterranean mixing in permeable media of sea water and ground water is studied. The model for this mixing process which was suggested by C. K. Wentworth is adopted, but is soon discarded in favour of a more tractable formulation whose equivalence to the original model is established. The analysis is carried to the point where the determination of the salinity distribution of the ground water in a given subsoil requires only the solution of an elementary linear ordinary differential equation.

1. INTRODUCTION

The distribution of water in permeable islands, e.g. the Hawaiian group, has the general configuration shown in figure 1. The depth of the lens-shaped region of fresh water (usually called the Guyben–Hertzberg lens) is nearly

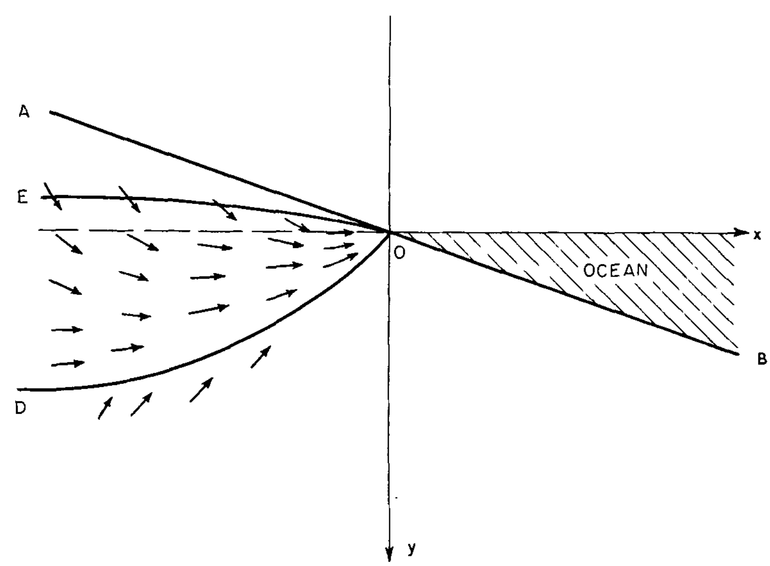

Figure 1. The distribution of ground water in a permeable island. The permeable material lies below *AOB*; the fresh water lies in the region between *EO* and *DO*; the salt water lies below *DOB*. A description of the salinity distribution near *DO* is the objective of this investigation. The arrows indicate qualitatively the velocity distribution associated with the 'steady' rainfall and run-off to the sea as described in § 4.

proportional to the one-half power of the distance from shore; its size is determined by the net water supply. However, the configuration is not stationary since there is an oscillatory vertical motion of the fluid in response to the tidal pressure excitation along OB. This motion implies, in the neighbourhood of the interface between the fresh water and the salt water, an invasion by the salt water of the cellular medium assigned to the fresh water, and vice versa; this leads to a dispersion of the material, so that the interface becomes diffuse. The purpose of this paper is to investigate quantitatively the structure of the transition zone separating the salt water from the fresh water.

In § 2 a physical model for the mixing process (due to C. K. Wentworth) is introduced; a continuum model which is thought to be equivalent is also suggested. The equivalence is established in § 3, by comparing solutions to certain preliminary problems. In § 4 a more realistic continuum formulation of the problem is given, and a solution is obtained; from this a more convenient equivalent mathematical model is established. In § 5 this last model is used to solve what I believe to be a satisfactory formulation of the problem of the structure of the mixing zone.

2. The mixing process

We shall adopt a mechanism due to C. K. Wentworth to explain the salinity distributions associated with the problems of interest. Imagine the porous structure to be a homogeneous array of communicating cells; a one-dimensional array like that of figure 2 suffices for our present purpose.

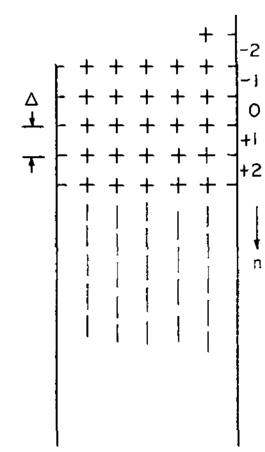

Figure 2. One-dimensional array of cells.

Denote by $S_{m,n}$ the salinity of the fluid in cell n at time t_m, and define a velocity $W(t) = M/\rho A f$, where M is the upward mass flow of fluid per unit time in the array of figure 2, A the cross-sectional area of the array, ρ the fluid density, and f the porosity (i.e. the fraction of the volume that can be occupied by fluid). According to this definition, W is the speed of a free surface advancing through the porous medium ahead of such a mass flow M. The cell height is denoted by Δ. $S_{m,n}$ may be taken to be either the volume fraction or the mass fraction of salt in the solution.

During the time interval t_m to t_{m+1}, a volume of fluid $(t_{m+1}-t_m)WA$ is convected from cells $n-1$ into cells n. We assume this process takes place without diffusion and that all the fluid in a cell n mixes thoroughly at time t_{m+1}. We choose the time interval sufficiently small so that no fluid from a cell $n-1$ passes into a cell $n+1$, i.e. $t_{m+1}-t_m < \Delta/W$.

Assuming that W is positive in the time $t_{m+1}-t_m$, cell n loses salinity $S_{m,n}(1-a)$ and gains salinity $S_{m,n-1}(1-a)$, so that

$$S_{m+1,n}-S_{m,n} = (1-a)[S_{m,n-1}-S_{m,n}], \tag{2.1}$$

where $1-a = |W|(t_{m+1}-t_m)/\Delta$.

When W is negative during the time interval, $S_{m,n-1}$ must be replaced by $S_{m,n+1}$. To include both cases, the difference equation governing the $S_{m,n}$ must be expressed as

$$S_{m+1,n}-S_{m,n} = \frac{(1-a)}{|W|}[(W+|W|)S_{m,n-1}-2|W|S_{m,n}+(|W|-W)S_{m,n+1}]. \tag{2.2}$$

Another model which has the same plausibility and which gives the same prediction, as we shall see in § 3, can be constructed by the following conventional limiting process. Divide each side of (2.2) by $t_{m+1}-t_m$, and let this difference tend to zero. We obtain

$$S_n'(t) = \{(|W|+W)S_{n-1}+(|W|-W)S_{n+1}-2|W|S_n\}/2\Delta. \tag{2.3}$$

Here $S_n(t)$ denotes the salinity in cell n at time t, and the prime denotes differentiation with regard to t. The corresponding formal limiting process wherein $\Delta \to 0$ would give a model in which the dispersive process had been eliminated. Consequently, we shall merely postulate a continuum model which 'resembles' (2.3) and establish its 'equivalence' with (2.3) by demonstrating that its interesting consequences are the same as those of (2.3). This continuum model, which is found by associating $S_{n+1}-2S_n+S_{n-1}$ with $\Delta^2 S_{yy}$, $S_{n+1}-S_{n-1}$ with $2\Delta S_y$, and $n\Delta$ with y, is

$$S_t = |W|\Delta S_{yy} - WS_y. \tag{2.4}$$

Here $S(t,y)$ is the salinity at time t and coordinate y, and the subscripts t and y denote partial differentiation with regard to those variables. Equations (2.2), (2.3) and (2.4) are each solved in § 3 for a given group of problems, and the equivalence of their predictions is established.

We here briefly discuss restrictions on the size of a. For a subsoil of complicated geometry this parameter must be determined experimentally, but for some special geometries we can deduce certain information about its value. If the cell array is composed, as in figure 3, of a tube with arbitrary vertical subdivisions, and if the time interval we adopt is the maximum possible for (2.2) to be valid, then $a = \frac{1}{2}$, since the flow at the very low

Reynolds numbers experienced in these phenomena would be paraboloidal in profile. Also, the volume of a paraboloid between the vertex and a section normal to the axis is one-half of that of the enclosing cylinder. If we use a shorter time interval, a is larger than $\frac{1}{2}$. If the cells are similar to those of figure 2, the flow from cell $n-1$ is likely to penetrate cell n in a comparatively slender filament, so again a will be larger than $\frac{1}{2}$. On the basis of such observations, it appears reasonable to expect that $a \geqslant \frac{1}{2}$. This is not a crucial point, however, since the determination of the effective cell size will be at least as important in determining the 'diffusion rate' of the phenomena.

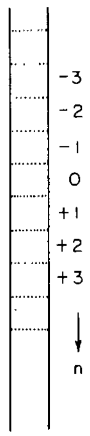

Figure 3. Tube with conceptual subdivision.

3. The solutions of the diffusion equations

The treatment of equation (2.2) with $-\infty < n < \infty$, and with $S_{0,n}$ given, is most readily carried out when we introduce a generating function

$$g_m(z) = \sum_{n=-\infty}^{\infty} S_{m,n} z^n,$$

where the expansion is a Laurent expansion. To solve (2.2) we multiply it by z^n and sum over n. The result is

$$g_{m+1}(z) = \left[1-(1-a)(1-z)\frac{|W|+W}{2|W|} - (1-a)(1-z^{-1})\frac{|W|-W}{2|W|}\right] g_m(z). \tag{3.1}$$

If m_1 is the number of time intervals during which $W > 0$, and $m_2 = m - m_1$, the solution of (3.1) is

$$g_m(z) = g_0(z)[1-(1-a)(1-z)]^{m_1}[1-(1-a)(1-z^{-1})]^{m_2}. \tag{3.2}$$

Since $S_{m,n}$ is the coefficient of the Laurent expansion of $g_m(z)$,

$$S_{m,n} = \frac{1}{2\pi i}\oint g_m(z) z^{-n-1}\, dz. \tag{3.3}$$

The contour encloses the origin and passes to the left of the point $z = 1$. For all the problems of interest here, $m \gg 1$; thus, an asymptotic evaluation of the integral (3.3) provides all the useful information. The solution depends on the initial conditions, i.e. on the values of $S_{0,n}$. No generality is lost if we take the initial distribution of salinity to be the step function which is unity for $n > 0$ and zero for $n < 0$, i.e. $g_0(z) = (1-z)^{-1}$.

With $g_0(z) = (1-z)^{-1}$,

$$S_{m,n} = \frac{1}{2\pi}\int_{-\pi}^{\pi} (1-z)^{-1}\exp\{-in\theta + m_1\log[1-(1-a)(1-z)] + \\ + m_2\log[1-(1-a)(1-z^{-1})]\}\,d\theta, \quad (3.4)$$

where $z(\theta) = e^{i\theta}$, and the path is indented to pass above the origin in the θ-plane. The saddle point of the exponent in (3.4) lies at

$$\theta = \theta_0 \sim -i[n-(1-a)(m_1-m_2)]/a(1-a)(m_1+m_2)$$

when

$$n-(1-a)(m_1-m_2) \ll m.$$

Since $\theta_0 \ll 1$ for such n, the second derivative of the exponent is closely approximated by $-a(1-a)m$. Thus, the integral which asymptotically approximates (3.4) is, with $\lambda = a(1-a)m$,

$$\begin{aligned} S_{m,n} &\sim \frac{i}{2\pi}\int_{-\infty}^{\infty} [\theta_0+(\theta-\theta_0)]^{-1}\exp\{\tfrac{1}{2}\lambda[\theta_0^2-(\theta-\theta_0)^2]\}\,d(\theta-\theta_0) \\ &= \tfrac{1}{2}\operatorname{erfc}\{-[n-(1-a)(m_1-m_2)]/[2a(1-a)(m_1+m_2)]^{1/2}\} \\ &= \tfrac{1}{2}\operatorname{erfc}\left\{-\left[y-\int_0^t W(\tau)\,d\tau\right]\Big/\left[2\Delta\int_0^t |W(\tau)|\,d\tau\right]^{1/2}\right\}. \end{aligned} \quad (3.5)$$

The final equality is obtained when we use the coordinate definitions which were introduced with (2.4). This result states that the mid-point of the transition zone translates with velocity $W(t)$, and that its breadth is proportional to $\left[\int_0^t W(\tau)\,d\tau\right]^{1/2}$.

The generating function $G(t,z)$ for the $S_n(t)$ in (2.3) has the form

$$G(t,z) = \sum_{-\infty}^{\infty} S_n(t)z^n = G(0,z)\exp\{-\alpha(1-z)-\beta(1-z^{-1})\}, \quad (3.6)$$

where $\alpha = (2\Delta)^{-1}\int_0^t (|W|+W)\,dt$ and $\beta = (2\Delta)^{-1}\int_0^t (|W|-W)\,dt$. From this, the integral representation for $S_n(t)$ is found to be precisely the limiting form of the right side of (3.4) when $a \to 1$, i.e.

$$S_n(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} (1-z)^{-1}\exp\{-\alpha(1-z)-\beta(1-z^{-1})-in\theta\}\,d\theta. \quad (3.7)$$

A discussion of the saddle point location and the second derivative of the exponent for $n \ll \alpha+\beta$ shows that (3.7) leads again to (3.5). For large n, this could not be because the signal speed of the model of (2.2) is finite and that of (2.6) is not.

We now denote the solution of (2.4) by $S(t,y)$ and its Fourier transform by $\sigma(t,\eta)$, i.e.

$$\sigma(t,\eta) = \int_{-\infty}^{\infty} S(t,y)\exp[-i\eta y]\,dy. \tag{3.8}$$

By the use of conventional techniques on (2.4) we obtain

$$\sigma_t(t,\eta) = [-iW\eta - |W|\Delta\eta^2]\sigma(t,\eta). \tag{3.9}$$

The solution, with $S(0,y)$ equal to the unit step function, is

$$\sigma = \sigma(0,\eta)\exp\{-(\alpha+\beta)\Delta^2\eta^2 - i(\alpha-\beta)\Delta\eta\},$$

where α and β are defined following (3.6), and $\sigma(0,\eta) = (i\eta)^{-1}$. Thus,

$$S(t,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty} (i\eta)^{-1}\exp[-(\alpha+\beta)\Delta^2\eta^2 - i(\alpha-\beta)\eta\Delta + i\eta y]\,d\eta. \tag{3.10}$$

This time equation (3.5) rigorously defines the function implied by (3.10).

The foregoing comparisons could be extended to more general initial considerations, but no further justification for the use of the continuum model seems necessary.

4. Mixing with a spacially varying velocity field

The Guyben–Hertzberg lens of fresh water is maintained in the presence of rainfall and the accompanying drainage of fluid to the sea. Thus, the velocity distribution above the salt–fresh water interface must resemble that of figure 1. Because of the diffuse character of the interface, some of the run-off must be salty, whereas the vertical intake is fresh water. If there were no motion other than the tidal fluctuations below the interface, the concentration at any point (for any initial solute distribution in dynamic equilibrium) would diminish with time. However, such a diminished concentration could not continue to be a hydrodynamic equilibrium configuration, and more of the denser salt water would have to be supplied by the sea to restore equilibrium. Thus, the only acceptable velocity distribution both above and below the interface is depicted in figure 1. It is easier to generalize the diffusion equation (2.4) to deal with an appropriate one-space variable problem and then tackle the problem with the flow field of figure 1, than to deal immediately with the latter problem. The appropriate one-dimensional problem is one in which the vertical velocity is given by

$$W = v\cos\omega t - \epsilon y. \tag{4.1}$$

The corresponding physical situation is one in which this velocity distribution occurs in a vertical column of cells and a flux of fluid emerges laterally from these cells with a mass conserving distribution. With this velocity distribution, (2.4) becomes

$$S_t = (\epsilon y - v\cos\omega t)S_y + |\epsilon y - v\cos\omega t|\Delta S_{yy}. \tag{4.2}$$

Because of the variable coefficients and the non-analytic character of the coefficient of S_{yy}, rigorous solutions to this differential equation would be very difficult to obtain. We must simplify it, and the motivation for the simplification can be better appreciated if we first anticipate some physical features of the phenomenon. With the ϵy term absent the solution of (4.1) is given by (3.5) and is

$$S(t,y) = \tfrac{1}{2}\operatorname{erfc}\{-[y-(v/\omega)\sin\omega t]/[4t\pi^{-1}+2P(t)]^{1/2}\} \tag{4.3}$$

where $P(t)$ has period ω and average zero. Hence S has an error function distribution with a breadth proportional to $t^{1/2}$ which translates at speed $v\cos\omega t$ and about which fluctuations occur at frequency ω. We can expect that the principal effect of the ϵy velocity contribution will be to squeeze the mixing zone into a narrower configuration, and that the other features will remain qualitatively unaltered.

In order to simplify (4.2), we note that the term $|\epsilon y - v\cos\omega t|$ is, for all $\epsilon y \ll v$, very well approximated by $|v\cos\omega t|$, except when t is very close to $\tfrac{1}{2}(2n+1)\pi$. Since almost no mixing occurs during such time intervals, the final term of (4.2) can be replaced by $\Delta|v\cos\omega t|S_{yy}$ with negligible loss of accuracy. When we introduce the dimensionless quantities $\tau = \omega t$, $x = y(\epsilon/v\Delta)^{1/2}$, $\alpha^2 = \epsilon\Delta/v$, $\beta^2 = \omega^2\Delta/\epsilon v$, equation (4.1) becomes

$$\beta S_\tau = (\alpha x - \cos\tau)S_x + a|\cos\tau|S_{xx}. \tag{4.4}$$

We also write $\xi = x - \beta^{-1}(1+N^{-2})^{-1}\sin[\tau + \arctan N^{-1}]$ with $N = \beta/\alpha$. The transformation to ξ allows for the oscillating translation of the salinity distribution. Equation (4.4) becomes

$$NS_\tau = \xi S_\xi + |\cos\tau|S_{\xi\xi}. \tag{4.5}$$

Typical estimates of the parameters are $\beta = 10^2$, $\alpha = 10^{-2}$, $N = 10^4$ (hence, terms of order N^{-1} in the definition of ξ can be dropped with no important loss of accuracy). Since we are primarily interested in the salinity distribution after a long time with the boundary conditions $S \to 1$ as $\xi \to \infty$, $S \to 0$ as $\xi \to -\infty$, we again use the initial condition at $t = 0$ that $S = 0$ for negative ξ and $S = 1$ for positive ξ.

If we define the Fourier transform

$$\bar{S}(\sigma,\tau) = \int_{-\infty}^{\infty} e^{-i\eta\xi} S(\xi,\tau)\, d\xi \tag{4.6}$$

with $\sigma = i\eta$, (4.5) becomes

$$N\bar{S}_\tau = -(\sigma\bar{S})_\sigma + |\cos\tau|\sigma^2\bar{S}. \tag{4.7}$$

Furthermore, with $z = \log\sigma$ and

$$u = \tau - z/N, \qquad v = \tau + z/N, \tag{4.8}$$

(4.7) can be written

$$2N\bar{S}_v + [1 - |\cos\tfrac{1}{2}(u+v)|e^{(v-u)/N}]\bar{S} = 0. \tag{4.9}$$

The initial condition at $\tau = 0$ becomes $\bar{S} = 1/\sigma$ when $u+v = 0$. Equation (4.9) is a first-order linear equation which is solved by using an integrating factor. That solution which is consistent with the foregoing initial conditions and in which the substitutions (4.8) have been used is

$$\bar{S}(\sigma, \tau) = \sigma^{-1}\exp[\sigma^2/H(\tau)], \tag{4.10}$$

where

$$H(\tau) = \pi^{-1}(1-e^{-2\tau/N}) + N^{-1}[P(\tau) - e^{-2\tau/N}P(0)]. \tag{4.11}$$

The function $P(\tau)$ is a periodic function whose Fourier series is

$$P(\tau) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{2\cos(2n\tau - \phi_n)}{\pi(4n^2-1)(n^2+N^{-2})^{1/2}}, \tag{4.12}$$

where $\phi_n = \arctan(nN)$. Note that $P(\tau) < \sum 2/(3\pi n^3) < 1$.

Equation (4.10) can be inverted to give $S(\xi, \tau)$. The result is

$$S(\xi, \tau) = \tfrac{1}{2}(1 + \operatorname{erf}\{\tfrac{1}{2}\xi[H(\tau)]^{1/2}\}). \tag{4.13}$$

Recalling that $P(\tau) < 1$ and that a typical value of N is 10^4, it is clear that after a large time

$$\begin{aligned} S(\xi, \tau) &\sim \tfrac{1}{2}[1 + \operatorname{erf}\{(\tfrac{1}{2}\pi)^{1/2}\xi\}] \\ &= \tfrac{1}{2}[1 + \operatorname{erf}\{(\tfrac{1}{2}\pi)^{1/2}[x - \beta^{-1}\sin\tau]\}. \end{aligned} \tag{4.14}$$

It can now be seen that when N is large and when only the results for large τ are wanted, (4.5) can be replaced by one in which the 'diffusion coefficient' $|\cos\tau|$ is replaced by its average value $2/\pi$. Note, however, that this asymptotic behaviour is reached only after dimensionless times τ of the order of N. To the accuracy with which we can now estimate N, this may be a few decades. Thus, when a large change in ground water usage habits occurs (e.g. irrigation), all the the effects may not be evident for several years. Taking advantage of the foregoing, (4.2) is replaced by

$$S_t = (2/\pi)v\Delta S_{yy} + (\epsilon y - v\cos\omega t)S_y. \tag{4.15}$$

In the system in which the coordinates are t and $y' = y - (v/\omega)\sin(\omega t - \phi)$, this equation becomes

$$S_t = (2/\pi)v\Delta S_{y'y'} + \epsilon y' S_{y'}. \tag{4.16}$$

Finally, for very large times, the solution becomes time independent, and S_t in (4.16) can be replaced by zero to give

$$(2/\pi)v\Delta S_{y'y'} + \epsilon y' S_{y'} = 0. \tag{4.17}$$

5. A two-dimensional model for mixing in the Guyben–Hertzberg lens

The results of §4 imply that in a one-dimensional array of cells in which the fluid velocity distribution is $\epsilon y + v\cos\omega t$, the mixing which occurs is equivalent to that which would occur in a fluid of diffusivity $2v\Delta/\pi$ which moved with the velocity ϵy relative to a coordinate system translating at speed $v\cos\omega t$. We adopt this result in order to simplify the analysis of the problem in which the salinity depends on the two space variables x and y. The actual Guyben–Hertzberg lens has a mixing zone whose thickness decreases inland since the tidal response, and hence the effective diffusivity, decreases inland. We adopt coordinates such that $y = 0$ at the (translating) nominal interface position; this is essentially the y' of §4. The distance inland from the shoreline is $-x$. Let $V(x)\cos\omega t - y\epsilon(x)$ be the vertical velocity distribution; let $U(x)$ be the steady horizontal velocity; and let $\sigma(x)$ be the effective diffusivity $2V(x)\Delta/\pi$. In general $V(x)$ will increase with increasing x, and conservation of mass requirements imply that $U'(x) - \epsilon(x) = 0$. With this notation the two-dimensional generalization of (4.17) is

$$U(x)S_x - \epsilon(x)yS_y = \sigma(x)S_{yy}. \tag{5.1}$$

The solution of this equation under the now familiar boundary conditions, $S \to 1$ as $y \to \infty$, $S \to 0$ as $y \to -\infty$, is of the form

$$S = \tfrac{1}{2}[1 + \operatorname{erf}\{y/h(x)\}], \tag{5.2}$$

and substitution of this into (5.1) shows that

$$[h^2(x)]' + (2\epsilon/U)h^2 = 4\sigma/U. \tag{5.3}$$

For given $\epsilon(x)$, $U(x)$, $\sigma(x)$, and for a given $h(x_0)$ this equation can be integrated easily.

Preliminary experiments on Maui island indicate that the physical facts are consistent with our predictions; however, there are too many guesses involved in choosing Δ, ϵ, σ, V to make any serious claims until more extensive measurements yield values for these quantities.

6. The effects of molecular diffusion

Once fluid has been transported across the passage connecting two cells, the new fluid from cell $n-1$ and that already in cell n will be thoroughly mixed by the diffusion–mixing mechanism supplied by the irregular vortex motion in the cell. However, a few estimates are still needed to ascertain whether the transport of salt across the connecting passage is contributed primarily by convection or by molecular diffusion. To answer this question we first note that the effective diffusivity of the model of equation (4.1) is $2V\Delta/\pi$. (This is implied by the solutions which we have presented.)

We now turn to the molecular diffusion process. We imagine the passages between cells to be holes of area a^2, and assume $a \ll \Delta$. The salinity gradient near and in the passage will be characterized by the quantity $\delta S/a$ (δS is $S_n - S_{n-1}$), and the total rate of flow of salt per unit time from cell $n-1$ into cell n by diffusion (without convection) is characterized by $\nu a \delta S$, where ν is the molecular diffusivity. Without a barrier, the salinity gradient is of order $\delta S/\Delta$, and the diffusive transport of salt across the area Δ^2 is of order $\nu \delta S \Delta$. The latter rate would be predicted by a conventional diffusion equation with diffusivity ν. The former rate $\nu \delta S a$ would be predicted by a conventional diffusion equation with diffusivity $\nu a/\Delta$. We call this latter quantity the effective diffusivity, i.e. the diffusion coefficient which must be used in the diffusion equation to obtain the proper diffusive transport for problems having the cellular geometry. The diffusivity which is associated with the mixing mechanism discussed in the preceding sections of this paper is $2V\Delta/\pi \doteqdot V\Delta$. Thus, the relative effectiveness of these two transport mechanisms is given by the ratio $\nu a/V\Delta^2$.

The value of a for a material of known permeability can be estimated in the following manner. The energy dissipation rate E per cell (assuming two holes to a cell) is of the order $\mu(V\Delta^2/a^3)^2a^3$, where μ is the fluid viscosity, $V\Delta^2/a^2$ is the characteristic velocity near the holes and $V\Delta^2/a^3$ is the velocity gradient near the holes. Furthermore, the definition of the permeability k states that $|\text{grad}\,p| = \mu fV/k$, where f is the porosity, and $\Delta^3V|\text{grad}\,p|$ is the energy loss rate per cell since the pressure drop across the cell $\Delta|\text{grad}\,p|$ acts on a fluid area Δ^2 against a velocity V. Equating these two estimates of E, we obtain $\mu V^2\Delta^4a^3 = \Delta^3V^2\mu f/k$ or $kf^{-1} = a^3/\Delta$. In the Hawaiian volcanic structure, kf^{-1} is of the order 10^{-5} cm^2; and near the test wells on Maui, Δ can be estimated at 0·5 cm, $\nu = 1\,\text{cm}^2/\text{day}$, $V = 30\,\text{cm/day}$. Using these figures, the effective molecular diffusivity $\nu a/\Delta$ is approximately 1/30 cm^2/day, whereas $2V\Delta/\pi = 15$. Thus, if the values of the parameters involved in any particular problem lie anywhere near the foregoing, the molecular diffusivity cannot play a competitive role in the determination of the salinity distribution.

7. Conclusion

In view of the lack of quantitative information regarding cell size and detailed velocity fields in permeable islands, we cannot quantitatively describe the salinity distribution in particular areas. The observations that we do offer are that the discretized model of equation (2.2), the continuum model of (2.4) and the 'intermediate' model of (2.3) all imply the same quantitative predictions regarding the structure of the brackish zone for which the tidal fluctuations provide the mixing mechanism. The more highly simplified models introduced in §4 and §5 to deal with more complicated flow configuration are equally appropriate for the prediction of such salinity distributions. It is also clear that the predictions of the last of these can be evaluated without difficulty for any given porous medium properties and a given velocity field.

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